EXAM PAPERS: 1976 – ~~1983~~ – 1982

LEAVING CERTIFICATE EXAMINATION, 1976

APPLIED MATHEMATICS – HIGHER LEVEL

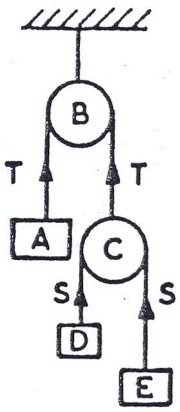
FRIDAY, 25 JUNE – MORNING, 9.30 to 12

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of $g$ to be 9·8 metres/second². $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. Show that, if a particle is moving in a straight line with constant acceleration $k$ and initial speed $u$, the distance travelled in time $t$ is given by $s = ut + \frac{1}{2}kt^2$. Two points $a$ and $b$ are a distance $l$ apart. A particle starts from $a$ and moves towards $b$ in a straight line with initial velocity $u$ and constant acceleration $k$. A second particle starts at the same time from $b$ and moves towards $a$ with initial velocity $2u$ and constant deceleration $k$. Find the time in terms of $u$, $l$ at which the particles collide, and the condition satisfied by $u, k, l$ if this occurs before the second particle returns to $b$.

2. A particle is projected upwards with a speed of 35 m/s from a point 0 on a plane inclined at 45° to the horizontal. The plane of projection meets the inclined plane in a line of greatest slope and the angle of projection, measured to the inclined plane, is $\phi$. Write down the velocity of the particle and its displacement from 0, in terms of $\vec{i}$ and $\vec{j}$, after time $t$ seconds. If the particle is moving horizontally when it strikes the plane at $q$ prove that $\cot\phi = 3$ and calculate $|oq|$.

3. The diagram shows a light inelastic string, passing over a fixed pulley B, connecting a particle A of mass 3M to a light movable pulley C. Over this pulley passes a second light inelastic string to the ends of which are attached particles D, E of masses 2M, M respectively. Show in separate diagrams the forces acting on A, D and E. Write down the three equations of motion involving the tensions T, S in the strings, the acceleration of A and the common acceleration of D, E *relative* to C. Show that T = 2S = 48Mg/17.




4. A light smooth ring of mass M is threaded on a smooth fixed vertical wire and is connected by a light inelastic string, passing over a fixed smooth peg at a distance $l$ from the wire, to a particle of mass 2M hanging freely. The system is released from rest when the string is horizontal. Explain why the conservation of energy can be applied to the system. If the ring descends a distance of $x$ while the particle rises through a distance $y$ show that

$$x^2 = y^2 + 2ly \text{ and } (l + y)\dot{y} = x\dot{x}$$

where $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$ are the speeds of the ring and particle respectively. Find $\dot{x}$ when (i) $x = l$ and (ii) when $x = 4l/3$.

5. State the laws governing the oblique collision of elastic spheres.
A sphere of mass M moving with speed $u$ collides obliquely with a second smooth sphere at rest. The direction of motion of the moving sphere is inclined at 45° to the line of centres at impact, and the coefficient of restitution is $\frac{1}{2}$. After impact the directions of motion of the spheres are at right angles.
Find the mass of the second sphere in terms of M, and the velocities of the two spheres after impact in terms of $u$. Hence show that one quarter of the kinetic energy is lost.

6. Two uniform rods $ab$, $bc$ of lengths $2l$, $2r$ and of weights 2W, 3W respectively are smoothly hinged together at $b$. They stand in equilibrium in a vertical plane with the end $a$ resting on rough horizontal ground and the end $c$ resting against a smooth vertical wall. The point $a$ is farther from the wall than $b$ and the rods $ab$, $bc$ are inclined at angles $\alpha$, $45°$ respectively to the horizontal where $\alpha > 45°$. Show in separate diagrams the forces acting on each rod. By considering separately the equilibrium of the system $abc$ and of the rod $bc$, find the coefficient of friction at $a$ and show that $\tan\alpha = 8/3$.

7. Define simple harmonic motion.

A particle of mass 2 kg is attached to the ends of two light elastic strings, each of natural length 1 m and elastic constant 49 N/m. The other ends of the two strings are attached to two fixed points $a$ and $b$ in the same vertical line, where $a$ is 4 m above $b$. The particle is released from rest from the midpoint of $ab$. By considering the forces acting on the particle when it is $x$ metres from $a$, where $2 < x < 2{\cdot}4$, show that it is moving with simple harmonic motion. Find the least time taken for the particle to reach the point $x = 2{\cdot}3$, and find its speed there.

8. A pendulum of a clock consists of a thin uniform rod $ab$ of mass M and length $6l$ to which is rigidly attached a uniform circular disc of mass 4M and radius $l$ with the centre of the disc being at the point $c$ on $ab$ where $bc = l$. Using the parallel axes theorem for the disc, show that the moment of inertia of the pendulum about an axis at $a$ perpendicular to the plane of the disc is $114Ml^2$.

The pendulum is free to oscillate in a vertical plane about such a fixed horizontal axis at $a$. It is released from rest with $ab$ horizontal. Find the speed of $b$ when $ab$ is vertical.

9. An atomic nucleus of mass M is repelled from a fixed point $o$ by a force $Mk^2x^{-5}$, where $x$ is the distance of the nucleus from $o$ and $k$ is a constant. It is projected directly towards $o$ with speed $\frac{2k\sqrt{3}}{d^2}$ from a point $a$ where $|oa| = d$. Find the speed of the nucleus when it reaches the midpoint of $oa$ and find how near it gets to $o$.

10. (i) Using Taylor's theorem (Mathematics Tables, p.42) find the first two terms in the Taylor series for $e^{x^2}$ in the neighbourhood of $x = 0$, i.e. the Maclaurin series for $e^{x^2}$.

(ii) State Archimedes principle for a body wholly or partly immersed in a liquid.

A uniform thin rod is of length $2a$, of weight 4W and specific gravity $4/9$. The rod rests in equilibrium in an inclined position partly immersed in water with its lower end freely pivoted to a fixed point at a depth $\frac{2a}{3}$ below the surface of the water. Show in a diagram the forces acting on the rod and calculate the inclination of the rod to the vertical.

LEAVING CERTIFICATE EXAMINATION, 1977

APPLIED MATHEMATICS - HIGHER LEVEL

FRIDAY, 24 JUNE - MORNING, 9.30 to 12

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of $g$ to be 9·8 metres/second$^2$. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A car starts from rest at P and moves with constant acceleration $k$ metres/second$^2$. Three seconds later another car passes through P travelling in the same direction with constant speed $u$ metres/second, where $u > 3k$. Draw a velocity/time graph for the two cars, using the same axes and the same scales.

Hence, or otherwise, show that the second car will just catch up on the first if $u = 6k$, and that it will not catch up on it if $u < 6k$.

If $u > 6k$, find the greatest distance the second car will be ahead of the first.

2. Explain, with the aid of a diagram, what is meant by the relative velocity of one body with respect to another.

To a cyclist riding North at 7 m/s the wind appears to blow from the North-West. To a pedestrian walking due West at 1 m/s the same wind appears to come from the South-West. Find the magnitude and direction of the velocity of the wind, by expressing it in the form $u\vec{i} + v\vec{j}$ or otherwise.

3. A particle is projected up a plane which is inclined at an angle $\alpha$ to the horizontal, where $\tan\alpha = \dfrac{1}{2\sqrt{3}}$. The direction of projection makes an angle of 60° with the inclined plane. The plane of projection is vertical and contains the line of greatest slope. Show that the particle strikes the inclined plane at right angles.

Verify that the total energy of the particle at the moment of striking the plane is the same as when the particle is first projected.

4. A mass of 2 kg is lying on a rough plane inclined at 60° to the horizontal, coefficient of friction $\frac{1}{2}$. The 2 kg mass is connected, by a light inextensible string passing over a smooth fixed pulley at the top of the plane, to a mass of 5 kg hanging freely. When the system is set free the 5 kg mass moves downwards.

Show in separate diagrams the forces acting on each mass, and calculate the common acceleration.

If a mass of 12 kg were used instead of the 2 kg mass, show by considering the forces acting that it would not move up the plane or down the plane.

5. A pump raises water from a depth of 5 m and discharges it horizontally through a nozzle of diameter 0·14 m at a speed of 10 m/s. Calculate

(i) the mass of water raised per second,

(ii) the kinetic energy given to this mass,

(iii) the power at which the pump is working.

If the water strikes a fixed vertical wall directly in front of the nozzle, find the force exerted by the water on the wall, on the assumption that no water bounces back.

[Mass of 1 m$^3$ of water is 1000 kg. Take $\pi = \frac{22}{7}$ ]

6. One end of a uniform ladder of weight $W$ rests against a smooth vertical wall and the other rests on rough horizontal ground so that it makes an angle $\tan^{-1}\frac{2}{3}$ with the horizontal. Show that the ladder will start to slip outwards if the coefficient of friction $\mu$ is less than $\frac{3}{4}$.

When $\mu = \frac{1}{2}$ the ladder is just prevented from slipping by a vertical string attached to the ladder at a point $\frac{1}{3}$ of its length from the top. Calculate the tension in the string in terms of $W$.

7. A smooth sphere of mass 3 kg moving at $\sqrt{29}$ m/s collides with a second sphere of mass 6 kg moving at 5 m/s. The directions of motion of the spheres make angles of $\tan^{-1}\frac{2}{5}$ and $\tan^{-1}\frac{4}{3}$, respectively, with the line of centres, both angles being measured in the same sense. The coefficient of restitution is $\frac{3}{4}$.

Find the speeds and directions of motion of the spheres after impact and calculate the kinetic energy lost in the collision.

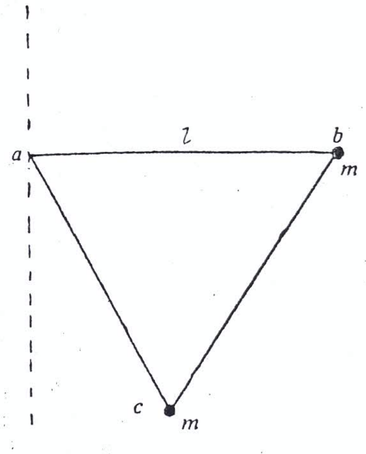
8. The position vector of a particle moving in a circle of radius $r$ with constant angular velocity $\omega$ can be expressed in the form

$$r \cos \omega t \,.\, \vec{i} + r \sin \omega t. \vec{j}.$$

Find the acceleration of the particle and show that it is directed towards the centre.

Three light rods $ab$, $bc$, $ca$, each of length $l$, are freely jointed to form a triangle $abc$. Two particles of mass $m$ are attached, one at $b$ and one at $c$. The system rotates about a vertical axis through $a$ with constant angular velocity $\omega$ such that $ab$ is horizontal and $c$ is vertically below $ab$. (see diagram)

Show in separate diagrams the forces acting on the particles (the forces exerted by the rods act along the rods). Calculate the forces in the rods and prove that $\omega^2 = 2\sqrt{3}\, g/l$.




9. For a compound pendulum (a rigid body performing small oscillations in a vertical plane about a horizontal axis) prove that the periodic time $T$ is given by

$$T = 2\pi \sqrt{\frac{I}{mgh}}$$

where $m$ is the mass of the pendulum, $I$ the moment of inertia about the axis, and $h$ the perpendicular distance from the centre of gravity to the axis.

If the compound pendulum is a uniform rod of length $2l$, show that $\frac{g}{4\pi^2 l} T^2 = \frac{h}{l} + \frac{1}{3}\frac{l}{h}$ and calculate the value of $\frac{h}{l}$ for which $T$ is a minimum.

10. State the Principle of Archimedes.

A tank contains a layer of water and a layer of oil of relative density 0·8. A uniform rod of relative density $\frac{7}{9}$ is totally immersed with one third of its volume in the water and two thirds in the oil. It is maintained in that position by two vertical strings attached to the ends of the rod and to the bottom of the tank.

Show in a diagram the forces acting on the rod and calculate the tensions in the strings in terms of $W$, the weight of the rod.

11. Answer any three of (a), (b), (c), (d) below.

(a) Using Taylor's theorem (Mathematics Tables, p. 42) find the first three terms in the Taylor series for $\frac{e^x}{1-x}$ in the neighbourhood of $x = 0$ i.e. the Maclaurin series for $\frac{e^x}{1-x}$.

(b) Determine if the series

$$(z-1) + \tfrac{1}{2}(z-1)^2 + \ldots + \tfrac{1}{n}(z-1)^n + \ldots\ldots$$

is absolutely convergent for $z = \frac{1}{4}(1+3i)$.

(c) Solve the differential equation

$$\frac{dy}{dx} = y \sin x$$

if $y = \sqrt{e}$ when $x = \frac{\pi}{3}$

(d) Solve the equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

if $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$.

# LEAVING CERTIFICATE EXAMINATION, 1978

## APPLIED MATHEMATICS – HIGHER LEVEL

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of $g$ to be 9·8 metres/second$^2$. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

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1. A driver starts from rest at P and travels with a uniform acceleration of $a$ m/s$^2$ for $T$ seconds. He continues with uniform velocity for $3T$ seconds, and then decelerates uniformly to rest at Q in a further $2T$ seconds. Express the distance PQ in terms of $a$ and $T$.

Another driver can accelerate at $2a$ m/s$^2$ and can decelerate at $4a$ m/s$^2$. Find, in terms of $T$, the least time in which this driver can cover the distance PQ from rest to rest

(i) subject to a speed limit of $3aT$ m/s,

(ii) subject to a speed limit of $5aT$ m/s.

2. A plane is inclined at an angle $\alpha$ to the horizontal. A particle is projected up the plane with initial velocity $u$ at an angle $\theta$ to the plane. The plane of projection is vertical and contains the line of greatest slope. Write down the displacement and velocity of the particle parallel and perpendicular to the plane at time $t$.

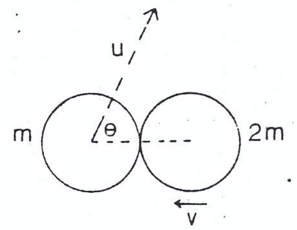
Show that the time taken by the particle to reach its maximum perpendicular height above the plane is half the time of flight up the plane.

When the particle is at its maximum perpendicular height above the plane, the distance travelled parallel to the plane is $\frac{2}{3}$ of the range up the plane. Show that in that case

$$\tan\theta \tan\alpha = \tfrac{2}{5}.$$

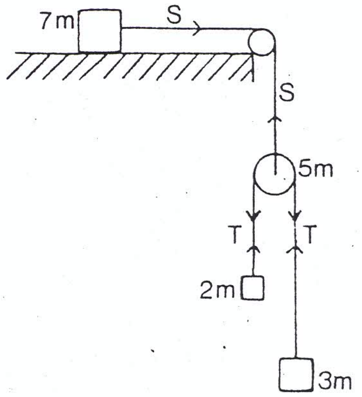
3. Two vectors $a\vec{i} + b\vec{j}$ and $c\vec{i} + d\vec{j}$ are at right angles. Write down the condition satisfied by the scalars $a, b, c, d$.

Two smooth spheres of masses $m$ and $2m$ and velocities $u$ and $v$, respectively, collide as shown in the diagram, where $\cos\theta = \frac{3}{7}$. The sphere of mass $m$ is deflected through an angle of 90° by the collision. If the coefficient of restitution is $\frac{3}{4}$, show that $v = \dfrac{11u}{7}$.




Find the direction of motion of the other sphere after the collision.

4. A body of mass $7m$ lies on a smooth horizontal table. It is connected by means of a light string passing over a smooth light pulley at the edge of the table, to a second smooth pulley of mass $5m$ hanging freely. Over this second pulley passes another light string carrying masses of $2m$ and $3m$ (see diagram).




Show in separate diagrams the forces acting on each of the masses. Write down the equations of motion involving the tensions $T$ and $S$ in the strings, the common acceleration $f$ of the $7m$ and $5m$ masses and the common acceleration $a$ of the $3m$ and $2m$ masses relative to the $5m$ mass.

Show that $f = \frac{7}{12} g$.

5. Two uniform rods AB and BC of equal length and of masses 5 kg and 3 kg, respectively, are freely hinged at B. AB and BC are in a vertical plane and the ends A and C are on a rough horizontal plane. The coefficient of friction between each rod and the plane is the same. Find the normal reactions at A and C.

The angle ABC is increased until one of the rods begins to slip. Show that slipping will first occur at C rather than at A.

Find the least value of the coefficient of friction if slipping has not occurred before $\angle ABC = 90°$.

6. Two small smooth rings A and B, each of mass $m$ are threaded on a fixed smooth horizontal wire. They are connected by means of two light inextensible strings AC and BC, each of length 2 metres, to a particle of mass $2m$ hanging freely at C. A, B, C are in the same vertical plane. The system is released from rest with the angle $\angle BAC = 30°$.

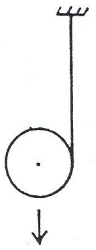
If A travels a horizontal distance $x$ while C falls a vertical distance $y$, show from geometry that $x^2 + y^2 - 2\sqrt{3}x + 2y = 0$.

By differentiating find $\dot{y}/\dot{x}$ in terms of $x$ and $y$, where $\dot{x}$ means $\dfrac{dx}{dt}$, and using the conservation of energy find $\dot{x}$ in terms of $y$.

Show that the velocity of A is $\sqrt{g(\sqrt{2}-1)}$ when $\angle BAC = 45°$.

7. Prove that the moment of inertia of a uniform circular disc about a perpendicular axis through its centre is $\frac{1}{2}mr^2$, where $r$ is the radius of the disc and $m$ is its mass.

A light string is wound around the rim of a uniform disc of radius $r$ and mass $m$. One end of the string is attached to the rim of the disc and the other end is attached to a fixed point above the disc, with the plane of the disc vertical (see diagram). When the disc is released from rest it falls vertically and the string unwinds.



If the disc falls a distance $x$ while it turns through an angle $\theta$, show that $x = r\theta$ and deduce that $\ddot{x} = r\dot{\omega}$, where $\omega$ is the angular velocity of the disc.

($\dot{x}$ means $\frac{dx}{dt}$, $\ddot{x}$ means $\frac{d^2x}{dt^2}$)

Using the principle of angular momentum, find the tension in the string and the vertical acceleration of the disc.

8. Solve the following differential equations:

(i) $\frac{dy}{dx} = \sqrt{1 - y^2}$ if $y = 0$ when $x = 1$

(ii) $\frac{d^2y}{dx^2} = -\frac{1}{y^3}$ if $\frac{dy}{dx} = 1$ and $x = \frac{1}{2}$ when $y = 1$.

A particle of mass $m$ is acted on by a force $2m/x^5$ directed away from a fixed point O, where $x$ is the distance of the particle from O. The particle starts from rest at a distance $d$ from O. Show that the velocity of the particle tends to a limit $1/d^2$.

9. (a) Two particles A and B are moving along two perpendicular lines towards a point O with constant velocities of 1·2 m/s and 1·6 m/s respectively. When A is 12 metres from O, B is 20 metres from O. Find the distance between them when they are nearest to each other.

(b) State the Principle of Archimedes.
A uniform circular cylinder of height $h$ and relative density $s$ floats with its axis vertical in a liquid of relative density $w$. Find the length of the axis of the cylinder immersed.
The cylinder is depressed vertically a further small distance $x$ and released. Show that it will perform simple harmonic motion, and find the period.

10. (a) A portion in the shape of an equilateral triangle is removed from a circular lamina of radius $r$. A vertex of the triangle was at the centre of the lamina and the sides of the triangle are of length $r$. Find the position of the centre of gravity of the remainder.

(b) A train of mass 300 tonnes is maintaining a steady speed of 10 m/s up an incline of 1 in 120 against frictional forces amounting to 40 kN. Calculate the power at which the engine is working. (1 tonne = 1000 *kg*)

(c) A corner on a level track has a radius of 100 m. Calculate the maximum speed at which a cyclist could take the corner if the coefficient of friction were $\frac{1}{4}$.

## LEAVING CERTIFICATE EXAMINATION, 1979

## APPLIED MATHEMATICS - HIGHER LEVEL

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of $g$ to be 9·8 metres/second$^2$. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. How may a velocity-time graph be used to find the distance travelled in a given time ?

An athlete runs 100 m in 12 seconds. Starting from rest, he accelerates uniformly to a speed of 10 m/s, and then continues at that speed. Calculate the acceleration.

A body starting from rest travels in a straight line, first with uniform acceleration $a$ and then with uniform deceleration $b$. It comes to rest when it has covered a total distance $l$. If the overall time for the journey is $T$, show that

$$T^2 = 2l\left(\frac{1}{a} + \frac{1}{b}\right)$$

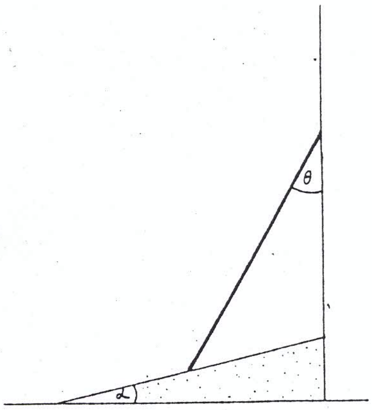
2. A ship A is travelling South-West at 10 knots. Another ship B is travelling at 20 knots in a direction 30° North of West. Draw a diagram to show the velocity of B relative to A.

Calculate the magnitude of the relative velocity, correct to the nearest knot, and its direction correct to the nearest degree.

By how much should A increase its speed, without changing direction, so that B would appear to A to be travelling due North ?

3. Explain the terms: coefficient of friction, angle of friction. What is the relationship between them ?

A uniform ladder of weight $W$ rests with one end against a smooth vertical wall and the other end on rough ground which slopes away from the wall at an angle $\alpha$ to the horizontal (see diagram). The ladder makes an angle $\theta$ with the wall.




Show that the reaction at the wall is $\frac{1}{2} W \tan \theta$.
If the ladder is on the point of slipping prove that

$$\tan \theta = 2 \tan (\lambda - \alpha)$$

where $\lambda$ is the angle of friction.

4. (a) A see-saw consists of a uniform plank freely pivoted at its mid-point on a support of vertical height $h$. It carries a mass $m$ at one end and a mass $2m$ at the other. The see-saw is released from rest with the mass $2m$ at its highest point. Find, in terms of $h$, the velocity with which the see-saw strikes the ground.

(b) State the Principle of Archimedes.
A uniform rod of length $l$ and relative density $s$ is freely hinged to the base of a tank containing a liquid to a depth $h$. The relative density of the liquid is $k$. As a result, the rod is inclined but not fully submerged. Derive an expression for the angle the rod makes with the horizontal.

5. A plane is inclined at an angle $\alpha$ to the horizontal. A particle is projected up the plane with a velocity $u$ at an angle $\theta$ to the plane. The plane of projection is vertical and contains the line of greatest slope.

(i) Show that the time of flight is $2u \sin \theta / g \cos \alpha$.

(ii) Prove that the range up the plane is a maximum when $\theta = \frac{1}{2}(\frac{\pi}{2} - \alpha)$ .

(iii) Prove that the particle will strike the plane horizontally if $\tan \theta = \sin \alpha \cos \alpha / (2 - \cos^2 \alpha)$.

6. (a) A particle, moving at constant speed, is describing a horizontal circle on the inside surface of a smooth sphere of radius $r$. The centre of the circle is a distance $\frac{1}{2}r$ below the centre of the sphere. Prove that the speed of the particle is $\frac{1}{2}\sqrt{6gr}$ .

(b) A conical pendulum consists of a light elastic string with a mass $m$ attached to it which is rotating with uniform angular velocity $\omega$ . The natural length of the string is $l$ and its elastic constant is $k$, i.e. a force $k$ produces unit extension. The extended length of the string is $l'$ and it makes an angle $\theta$ with the vertical. Prove that $k(l' - l) = m l' \omega^2$ and that

$$\cos \theta = \frac{g}{l\omega^2} - \frac{mg}{kl}$$

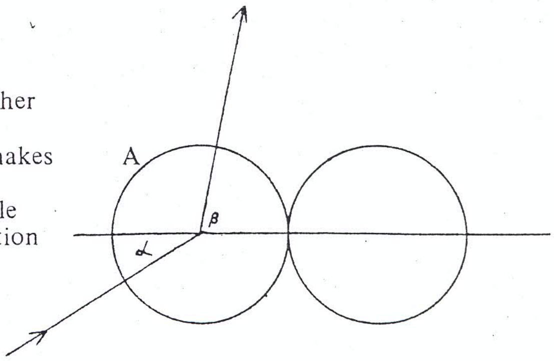
7. (a) Two smooth spheres of masses $m$ and $2m$ collide directly when moving in opposite directions with speeds $u$ and $v$, respectively. The sphere of mass $2m$ is brought to rest by the impact. Prove that

$$e = \frac{2v - u}{u + v}$$

where $e$ is the coefficient of restitution.

(b) A smooth sphere A collides obliquely with another smooth sphere of equal mass which is at rest. Before impact the direction of motion of A makes an angle $\alpha$ with the line of centres at impact (see diagram). After impact it makes an angle $\beta$ with that line. If the coefficient of restitution is $\frac{1}{2}$, prove that

$$\tan \beta = 4 \tan \alpha$$




8. (a) Define simple harmonic motion. Using the usual notation, show that the equation $v^2 = \omega^2 (a^2 - x^2)$ represents simple harmonic motion.

A body is moving with simple harmonic motion, of amplitude 5 m. When it is 4 m from the mid-point of its path its speed is 6 m/s. Find its speed when it is 2·5 m from the mid-point.

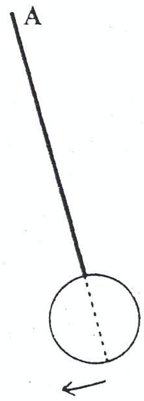
(b) A block rests on a rough platform which moves to and fro horizontally with simple harmonic motion. The amplitude is 0·75 m and 20 complete oscillations occur per minute. If the block remains at rest relative to the platform throughout the motion, find the least possible value the coefficient of friction can have.

9. State the theorem of parallel axes.

Prove that the moment of inertia of a uniform rod of mass $m$ and length $2l$ about a perpendicular axis through one of its ends is $4ml^2/3$.

A uniform rod of length $6l$ is attached to the rim of a uniform disc of diameter $2l$. The rod is collinear with a diameter of the disc (see diagram). The disc and the rod are both of mass $m$. Calculate the moment of inertia of the compound body about a perpendicular axis through the end A.

If the compound body makes small oscillations in a vertical plane about a horizontal axis through A, show that the periodic time is $2\pi\sqrt{123l/20g}$ .




10. (a) Solve the differential equation

$$x \frac{dy}{dx} = -y .$$

Hence or otherwise solve

$$x \frac{d^2y}{dx^2} = -\frac{dy}{dx}$$

where $y = 0$ when $x = 1$ and $y = 3$ when $x = e$.

(b) A body is moving in a straight line subject to a deceleration which is equal to $v^2/10$, where $v$ is the velocity. The initial velocity is 5 m/s. In how many seconds will the velocity of the body be 2 m/s and how far will it travel in that time ?

## LEAVING CERTIFICATE EXAMINATION, 1980

## APPLIED MATHEMATICS - HIGHER LEVEL

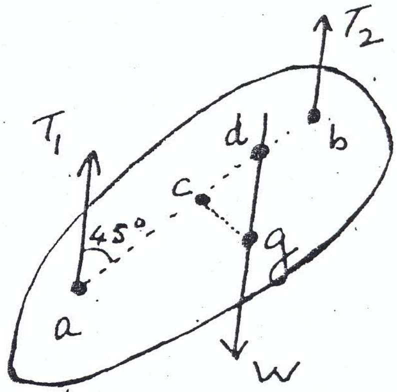
Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of $g$ to be 9·8 metres/second$^2$. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A boat has to travel by the shortest route to the point $4{\cdot}25\vec{j}$ km and then return immediately to its starting point at the origin. The velocity of the water is $(8\sqrt{2}\,\vec{i} - 8\sqrt{2}\,\vec{j})$ km/hour and the boat has a speed of 18 km/hour in still water.

If $a\vec{i} + b\vec{j}$ is the velocity of the boat on the outward journey, find $a$ and $b$ and the time taken for the outward journey, leaving your answer in surd form. Find, also, the time taken for the whole journey.

2. A body of weight $W$ is supported by two vertical inextensible strings at $a$ and $b$ as in diagram where $|ab| = 10$ cm. The tensions in the strings are $T_1$ and $T_2$ and the string of tension $T_1$ makes an angle of 45° with $ab$. The centre of gravity of the body is at $g$, the centre of $[ab]$ is $c$ and $cg \perp ab$.

Express $|cd|$ in terms of $W$ and $T_1$ and hence find the distance of $g$ from $ab$ in terms of $W$ and $T_1$.




3. A projectile is fired with initial velocity $\vec{u} = u\cos\alpha\vec{i} + u\sin\alpha\vec{j}$, where $\vec{i}$ is along the horizontal. A plane $P$ passes through the point of projection and makes an angle $\beta$ with the horizontal. If the projectile strikes the plane $P$ at right angles to $P$ after time $t$, show that

$$t = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

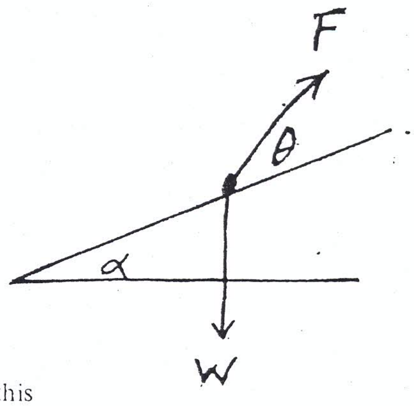
and deduce that $2\tan(\alpha - \beta)\tan\beta = 1$.

If $\alpha - \beta = \frac{\pi}{4}$, find in terms of $u$ and $g$ the range of the projectile along $P$.

4. State and prove the relationship between the coefficient of friction $\mu$ and the angle of friction $\lambda$.

The diagram shows a particle of weight $W$ on a rough plane making an angle $\alpha$ with the horizontal. The particle is acted upon by a force $F$ whose line of action makes an angle $\theta$ with the line of greatest slope. The particle is just on the point of moving up the plane. Draw a diagram showing the forces acting on the particle and prove that

$$F = \frac{W\sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$$

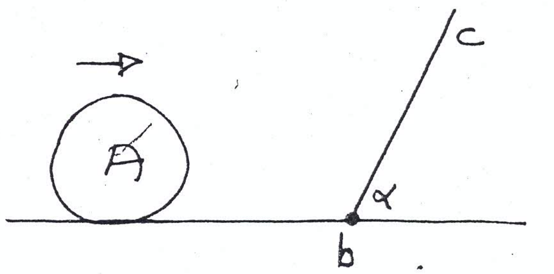



If the particle is just on the point of moving up the plane, deduce

(i) the force acting up along the plane that would achieve this
(ii) the horizontal force that would achieve it
(iii) the minimum force that would achieve it.

5. (a) Two imperfectly elastic spheres of equal mass moving horizontally along the same straight line impinge and, as a result, one of them is brought to rest. Show that whatever be the value of the coefficient of restitution, $e < 1$, they must have been moving in opposite directions.

(b) A sphere $A$ of mass $m$ kg moving with a speed $u$ m/s on a smooth horizontal table impinges on a smooth plane $bc$. This plane is inclined to the table at an angle $\alpha$ and the line of intersection of it with the table is at right angles to the direction of motion of the sphere. Write down the components of the velocity of $A$ perpendicular to the plane and parallel to the plane before impact and show that $eu\sin\alpha$ is the velocity of $A$ perpendicular to the plane after impact when $e$ is the coefficient of restitution between the sphere and the plane. Find the magnitude of the impulse due to the impact.



6. If a string whose elastic constant is $k$ is stretched a distance $x$ beyond its natural length, show that the work done is $\frac{1}{2}kx^2$.

A particle of mass $m$ is on a rough horizontal plane and is connected to a fixed point $p$ in the plane by a light string of elastic constant $k$. Initially the string is just taut and the particle is projected along the plane directly away from $p$ with initial speed $u$ against a constant resistance $F$. Find an expression for the distance $x$ travelled by the particle. Noting that the particle will just return to its point of projection if the potential energy at any point is equal to the work done up to that point in overcoming $F$, show that

$$kmu^2 = 8F^2.$$

7. Establish the moment of inertia of a uniform rod about an axis through its centre perpendicular to the rod.

State the parallel axes theorem.

A thin uniform rod of length $2l$ and of mass $m$ has a mass of $2m$ attached at its mid-point. Find the positions of a point in the rod about which the rod (with attached mass) may oscillate as a compound pendulum, having period equal to that of a simple pendulum of length $l$.

8. (a) A particle is moving in a straight line such that its distance $x$ from a fixed point at time $t$ is given by

$$x = r\cos\omega t.$$

Show that the particle is moving with simple harmonic motion.

A particle is moving in a straight line with simple harmonic motion. When it is a point $p_1$ of distance 0·8 m from the mean-centre, its speed is 6 m/s and when it is at a point $p_2$ of distance 0·2 m from the end-poisition on the same side of the mean-centre as $p_1$, its acceleration is of magnitude 24 m/s$^2$. If $r$ is the amplitude of the motion, show that

$$\frac{2}{3} = \frac{r - 0{\cdot}2}{r^2 - 0{\cdot}64}$$

and hence find the value of $r$.

Find also the period of the motion and the shortest time taken between $p_1$ and $p_2$ correct to two places of decimals.

9. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + \frac{2}{y^3} = 0$$

given that $\frac{dy}{dx} = \sqrt{2}$ and $x = \sqrt{2}$ when $y = 1$.

(b) A car starts from rest. When it is at a distance $s$ from its starting point, its speed is $v$ and its acceleration is $5 - v^2$. Show that

$$v\,dv = (5 - v^2)\,ds$$

and find as accurately as the tables allow its speed when $s = 1{\cdot}5$.

10. (a) A vessel is in the form of a frustum of a right circular cone. It contains liquid to a depth $h$ and at that depth the area of the free surface of the liquid is $\frac{1}{4}$ of the area of the base. Find in simplest surd form the ratio of the thrust on the base due to the liquid to the weight of the liquid.

(b) A piece of wood and a piece of metal weigh 14 N and 6 N, respectively. When combined together the compound body weighs 1·9 N in water. Given that the specific gravity of the metal is 10, find the specific gravity of the wood.

## LEAVING CERTIFICATE EXAMINATION, 1981

## APPLIED MATHEMATICS - HIGHER LEVEL

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of $g$ to be 9·8 metres/second$^2$. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A body starts from rest at $p$, travels in a straight line and then comes to rest at $q$ which is 0·696 km from $p$. The time taken is 66 seconds.

For the first 10 seconds it has uniform acceleration $a_1$. It then travels at constant speed and is finally brought to rest by a uniform deceleration $a_2$ acting for 6 seconds.

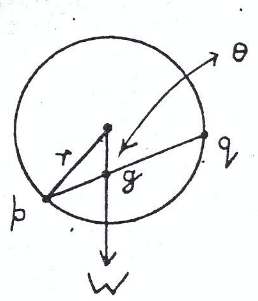
Calculate $a_1$ and $a_2$.

If the journey from rest at $p$ to rest at $q$ had been travelled with no interval of constant speed, but subject to $a_1$ for a time $t_1$ followed by $a_2$ for time $t_2$, show that the time for the journey is $8\sqrt{29}$ seconds.

2. If a body is in equilibrium under the action of two and only two forces, what can be deduced about the forces ?

If a body is in equilibrium under the action of three non parallel forces prove that their lines of action must be concurrent and that the forces may be represented in magnitude and direction by the sides of a triangle taken in order.

A straight, rigid non-uniform rod $[pq]$ of weight $W$ rests in equilibrium inside a smooth hollow sphere of radius $r$. The distances of its centre of gravity, $g$, from $p$ and $q$ are 4 and 6 cm respectively.




(i) Show that $\theta$, the angle the rod makes with the vertical, is given by

$$\cos^{-1}\left(\frac{1}{\sqrt{r^2 - 24}}\right).$$

(ii) Prove that the magnitude of the reaction at $p$ is given by

$$\frac{3\,r\,W}{5\sqrt{r^2 - 24}}.$$

3. Establish an expression, in terms of initial speed $u$ and angle of inclination to the horizontal $\alpha$, for the range of a projectile on a horizontal plane through the point of projection.

Deduce that the maximum range for a given $u$ is $u^2/g$.

A particle is projected at initial speed $u$ from the top of a cliff of height $h$, the trajectory being out to sea in a plane perpendicular to the cliff. The particle strikes the sea at a distance $d$ from the foot of the cliff. Show that the possible times of flight can be obtained from the equation

$$g^2 t^4 - 4(u^2 + gh)t^2 + 4(h^2 + d^2) = 0.$$

Hence, or otherwise, prove that the maximum value of $d$ for a particular $u$ and $h$ is

$$\frac{u\sqrt{u^2 + 2gh}}{g}.$$

4. (i) A sphere $A$, mass $m$, moving with velocity $2u$ impinges directly on an equal sphere $B$, moving in the same direction with velocity $u$. Show that the loss in kinetic energy due to the impact is

$$\frac{m\,u^2(1 - e^2)}{4}$$

where $e$ is the coefficient of restitution between the spheres.

(ii) If $B$ had been at rest and $A$ impinged obliquely, so that after impact, $A$ moved with velocity $2u$ in a direction making an angle of 30° with the line of centres of the spheres, show that the loss in kinetic energy is three times greater than in (i).

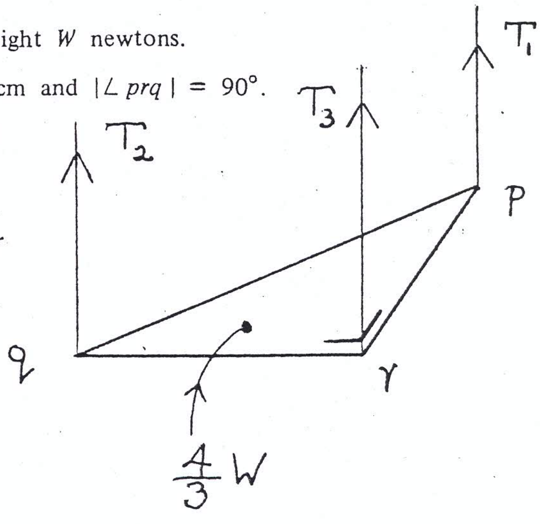
5. Prove that the centre of gravity of a uniform triangular lamina is at the point of intersection of its medians.

A uniform triangular lamina $pqr$ has weight $W$ newtons.

$|pq| = 5$cm, $|qr| = 4$cm, $|pr| = 3$cm and $|\angle prq| = 90°$.

The lamina is suspended in a horizontal position by three inextensible, vertical strings, one at each vertex.

A particle, of weight $\frac{4}{3}W$ newtons is positioned on the lamina 2cm from $pr$ and one cm from $qr$. Calculate the tension on each string in terms of $W$.



6. (a) Establish the formula $T = 2\pi\sqrt{\frac{l}{g}}$ for the periodic time of a simple pendulum of length $l$.

The length of a seconds pendulum ($T = 2$ secs) is altered so as to execute 32 complete oscillations per minute. Calculate the percentage change in length.

(b) A heavy particle is describing a circle on a smooth horizontal table with uniform angular velocity $\omega$. It is partially supported by a light inextensible string attached to a fixed point 0·1 metres above the table. Calculate the value of $\omega$ if the normal reaction of the table on the particle is half the weight of the particle.

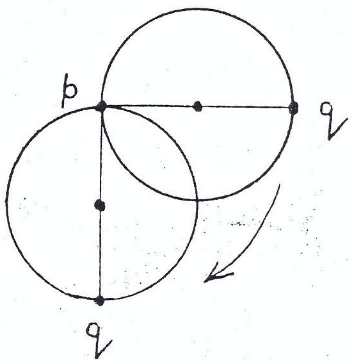
7. A uniform circular disk has mass $M$ and radius $R$. Prove that its moment of inertia, $I$, about an axis through its centre perpendicular to its plane is $\frac{1}{2}MR^2$.

Deduce the moment of inertia about an axis through a point on its rim perpendicular to its plane.

A uniform circular disk has mass $m$ and radius $r$. It is free to rotate about a fixed horizontal axis through a point $p$ on its rim perpendicular to its plane. A particle of mass 2 m is attached to the disk at a point $q$ on its rim diametrically opposite $p$.

The disk is held with $pq$ horizontal and released from rest. Find, in terms of $r$, the angular velocity when $q$ is vertically below $p$.

If the system were to oscillate as a compound pendulum, prove that it would have a periodic time equal to that of a simple pendulum of length $\frac{19r}{10}$.

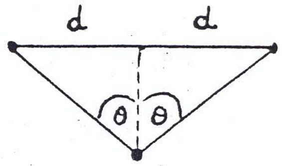



8. (a) A heavy particle is hung from two points on the same horizontal line and a distance $2d$ apart by means of two light, elastic strings of natural length $l_1$, $l_2$ and elastic constants $k_1$, $k_2$ respectively.

In the equilibrium position the two strings make equal angles $\theta$ with the vertical.

Prove that

$$\sin\theta = \frac{d\,(k_1 - k_2)}{k_1\, l_1 - k_2\, l_2}$$




(b) A horizontal platform, on which bodies are resting, oscillates vertically with simple harmonic motion of amplitude 0·2 m. What is the maximum integral number of complete oscillations per minute it can make, if the bodies are not to leave the platform ?

9. (a) A body is weighed in water and in each of two liquids of specific gravity 0·8 and 0·75. If the resulting weights, in order, are $w_1$, $w_2$, and $w_3$, verify that

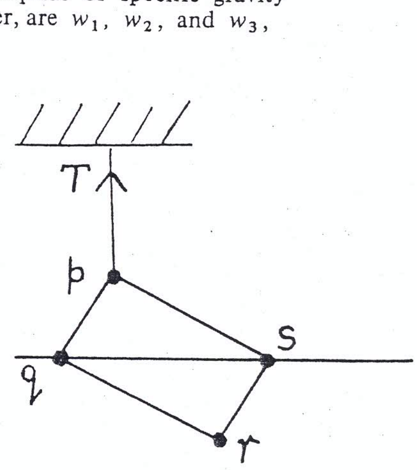
$$w_1 = 5w_2 - 4w_3 .$$

(b) A uniform rectangular board $pqrs$, $|pq| \neq |ps|$, hangs vertically in fresh water with the diagonal $qs$ on the surface. The board is held in that position by a vertical string at $p$.

(i) Show on a diagram all forces acting on the board.

(ii) Calculate the tension ($T$) in the string and the buoyancy force ($B$) in terms of $W$, the weight of the board.

(iii) Calculate the specific gravity of the board.



10. (a) Solve the differential equation

$$\frac{dy}{dx} + y^2 \cos^3 x = 0$$

given that $y = 2$ when $x = \pi/6$

(b) Find the general solution to

$$\frac{d^2y}{dx^2} = K \frac{dy}{dx}$$

where $K$ is a constant.

A particle moves in a straight line so that at any instant its acceleration is, in magnitude, half its velocity.

If its initial velocity is 3 m/s, find an expression for the distance it describes in the fifth second.

# LEAVING CERTIFICATE EXAMINATION, 1982

## APPLIED MATHEMATICS - HIGHER LEVEL

MONDAY, 28 JUNE - AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of $g$ to be 9·8 metres/second$^2$.

1. (a) A car $A$ passes a point $p$ on a straight road at a constant speed of 10 $m/s$. At the same time another car $B$ starts from rest at $p$ with uniform acceleration 2·5 m/s$^2$.

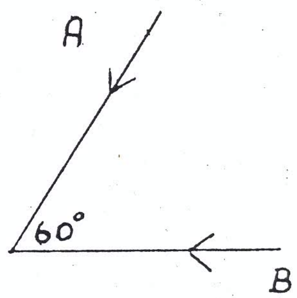
   (i) When and how far from $p$ will $B$ overtake $A$ ?

   (ii) If $B$ ceases to accelerate on overtaking, what time elapses between the two cars passing a point $q$ three kilometres from $p$ ?

   (b) A particle of mass 3 grammes falls from rest from a height of 0·4 m on to a soft material into which it sinks 0·0245 m. Neglecting air resistance, calculate the constant resistance of the material.

2. Two straight roads intersect at an angle of 60°. Cyclists $A$ and $B$ move towards the point of intersection at 30 km/h and 40 km/h, respectively. Calculate the velocity of $A$ relative to $B$.

If $A$ is 3·5 km and $B$ is 2 km from the intersection at a given moment, calculate the shortest distance between them in their subsequent motion.




3. A particle is projected with a speed of 10 m/s at an angle $\alpha°$ to the horizontal up a plane inclined at 30° to the horizontal. If the particle strikes the plane at right angles, show that the time of flight can be represented by the two expressions

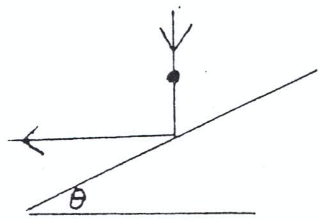
$$\frac{10\cos(\alpha - 30)}{g \sin 30} \quad \text{and} \quad \frac{20 \sin(\alpha - 30)}{g \cos 30}$$

Hence deduce a value for $\tan(\alpha - 30)$.

Calculate the range of the particle along the plane.

4. (a) A smooth sphere of mass 10 kg moving at 10 m/s impinges directly on another smooth sphere of mass 50 kg moving in the opposite direction at 5 m/s. If the coefficient of restitution is $\frac{1}{2}$, calculate the speeds after impact and the magnitude of the impulse during impact.

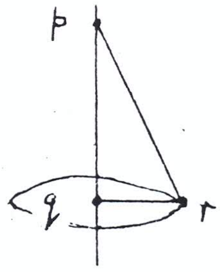
(b) A smooth metal sphere falls vertically and strikes a fixed smooth plane inclined at an angle of $\theta$ to the horizontal. If the coefficient of restitution is $\frac{2}{3}$ and the sphere rebounds horizontally, calculate the fraction of kinetic energy lost during impact.




5. (a) The diagram shows a string $prq$ which is fixed at $p$ and $q$ where $q$ is vertically below $p$. $r$ is a small ring threaded on the string which is made to rotate at an angular velocity, $\omega$ rad/s, in a horizontal circle, centre $q$, the string being taut.

If $|pq| = 0{\cdot}12$ m, $|pr| + |rq| = 0{\cdot}18$ m,

show that $\omega = \sqrt{294}$ rad/s.




(b) A small bead of mass $m$ is threaded on a smooth circular wire of radius $a$, fixed with its plane vertical. The bead is projected from the lowest point of the wire with speed $u$. Show that the reaction between the bead and the wire, when the radius to the bead makes an angle of 60° with the downward vertical, is

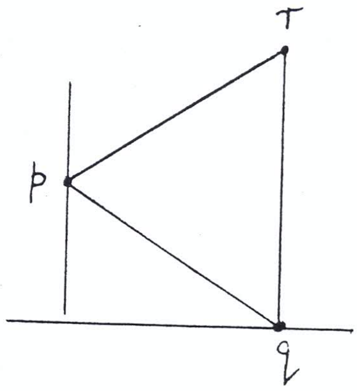
$$m\left[\frac{u^2}{a} - \frac{g}{2}\right]$$

6. Define (i) limiting friction, (ii) coefficient of friction.

A lamina of weight $W$ in the shape of a thin equilateral triangle $pqr$ is positioned vertically with the vertex $p$ against a smooth vertical wall, and $q$ on a rough horizontal floor. $[qr]$ is parallel to the wall.

Find in terms of $W$, the horizontal and vertical reactions at $q$.

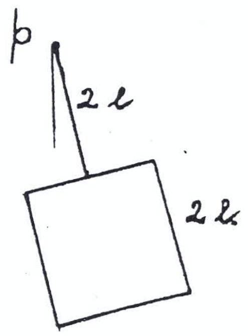
Find the least value of $\mu$, the coefficient of friction, so that slipping will not occur.




7. (i) Show that the moment of inertia of a uniform square lamina of side $2l$ and mass $m$ about an axis perpendicular to the lamina through its centre of mass is

$$\frac{2\,ml^2}{3}.$$

(ii) A thin uniform rod of length $2l$ and of mass $m$ is attached to the mid point of the rim of the square. Find the moment of inertia of the system about an axis through $p$ perpendicular to the common plane of the lamina and rod. [See Tables, P.40].




(iii) When this system makes small oscillations in a vertical plane about the axis through $p$, show that the period of the oscillations is

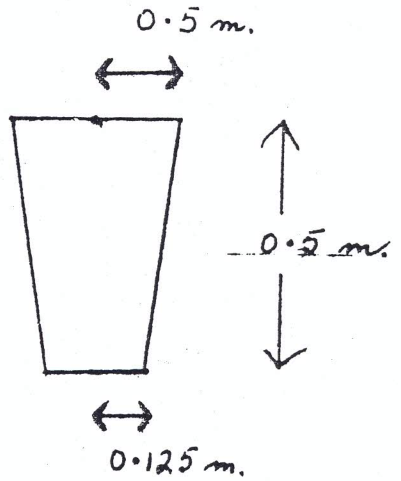
$$2\pi\sqrt{\frac{31\,l}{7g}}$$

8. Define simple harmonic motion.
The distance, $x$, of a particle from a fixed point, $o$, is given by

$$x = a \cos(\omega t + \alpha)$$

where $a$, $\omega$, $\alpha$ are positive constants.
Show that the particle is describing simple harmonic motion about $o$ and calculate $\omega$ and $\alpha$ if the velocity $v = -2a$ and $x = \frac{3a}{5}$ when $t = o$.
After how many seconds from the start of the motion is $x = o$ for the first time ? (See Tables P.8. Take $\pi = 3{\cdot}142$).

9. (a) A bucket has the form of a frustum of a right circular cone. When it is completely filled with water, find
(i) the pressure at a point on the base
(ii) the thrust, $T$, on the base
(iii) the ratio $\frac{\text{weight of water}}{T}$.




(b) A cubical block of wood of mass 50 kg floats in water with three quarters of its volume immersed.
In oil, when a mass of 10 kg is placed on the same block, it floats just totally immersed , the 10 kg mass being above the oil.
Find the specific gravity of the oil.

10. (a) Find the solution of the differential equation

$$(1 + x^3)\frac{dy}{dx} = x^2 y$$

when $y = 2$ at $x = 1$.

(b) Find the solution of the differential equation

$$\frac{d^2 s}{dt^2} = -\left(\frac{ds}{dt}\right)^2$$

when $\frac{ds}{dt} = 1$ at $t = 0$

and $s = 0$ at $t = 0$.

A particle moves in a straight line with acceleration equal to minus the square of its velocity. If its initial velocity is 1 m/s, calculate the distance travelled one second later.

## LEAVING CERTIFICATE EXAMINATION. 1983

APPLIED MATHEMATICS – HIGHER LEVEL

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of $g$ to be $9{\cdot}8$ metres/second$^2$.

1. A train of length 120 m has an acceleration of 1 m/s$^2$. It meets another train of length 80 m travelling on a parallel track in the opposite direction with an acceleration of $1{\cdot}5$ m/s$^2$. Their speeds at this moment are respectively 20 m/s and 25 m/s. Show, by diagrams, the positions of the trains just before meeting and immediately after passing.
Find the time taken for the trains to pass each other.
If one of the trains, by applying brakes, were to cause an increase of $12\frac{1}{2}$% in this time of passing, calculate to the nearest m/s$^2$ the decrease in its acceleration.